

Exercise set 3.

1. Let there be three individuals in an economy. All of them have utility function $u(x, y) = xy$. Individuals 1 and 2 have the same endowment $\omega_1 = \omega_2 = (1, 14)$, while individual 3 has endowment $\omega_3 = (27, 1)$. Find a core allocation that treats individuals 1 and 2 differently.

2. In a two-agent economy $u_1(x, y) = x + y$, $u_2(x, y) = \min\{x, y\}$, $\omega_1 = (2, 1)$ and $\omega_2 = (1, 2)$. Find the contract curve (Pareto efficient allocations), the core and Walras's equilibrium.

3. Suppose that two individuals have utility functions $u_1(x, y) = x + ay$ and $u_2(x, y) = x + by$, $a, b > 0$, and endowments $\omega_1 = (2, 1)$ and $\omega_2 = (1, 2)$. Find the contract curve (Pareto efficient allocations), the core and Walras's equilibrium for $a = b$, $a > b$ and $a < b$.

4. A Marshallian demand function $x(p)$ satisfies the gross substitute property, if $x_l(p) > x_l(\hat{p})$ holds for all $l \neq k$ whenever for two price vectors p and \hat{p} we have $p_k > \hat{p}_k$ for good k and $p_l = \hat{p}_l$ for all other goods. (Note: if each consumer's demand function satisfies the gross substitutes property then the aggregate excess demand function satisfies the property).

Show that if the aggregate excess demand function satisfies the gross substitutes property, then the economy has at most one Walrasian equilibrium.

5. MWG exercise 17.C.6.