

Exercise set 2.

1. Find the Walrasian equilibrium when agent 1 has utility function  $u_1(x, y) = x + y + xy$  and endowment  $\omega_1 = (1, 0)$ , while agent 2 has utility function  $u_2(x, y) = xy$  and endowment  $\omega_2 = (0, 1)$ .

2. Find the Walrasian equilibrium when agent 1 has utility function  $u_1(x, y) = \min\{2x, y\}$  and endowment  $\omega_1 = (2, 8)$ , while agent 2 has utility function  $u_2(x, y) = \min\{x, 3y\}$  and endowment  $\omega_2 = (6, 0)$ .

3. Consider an  $m$ -agent exchange economy  $\mathcal{E} = ((X_h, u_h, \omega_h)_{h \in \mathcal{H}})$  where consumption set is  $X_h = \mathbb{R}_+^l$ , endowment  $\omega_h$ , and each agent has a differentiable utility function. Assume that  $x^* \in \times_{h \in \mathcal{H}} X_h$  is a Pareto-optimal allocation. Use the Kuhn-Tucker condition and characterise  $x^*$  with Lagrange-multipliers.

4. (Technical) Consider an  $m$ -agent exchange economy  $\mathcal{E} = ((X_h, \succeq_h, \omega_h)_{h \in \mathcal{H}})$  where consumption set is  $X_h = \mathbb{R}_+^l$ , endowment  $\omega_h$ , and the preference relation complete, transitive, continuous and monotone for each agent. Show that the set of Pareto-optimal allocations is a compact subset of  $\mathbb{R}_+^{ml}$ .

5. There are two consumers are  $A$  and  $B$ .  $A$ 's endowment is given by  $\omega_A = (0, 0)$  and  $B$ 's by  $\omega_B = \left(\frac{1}{n}, 1 - \frac{1}{\sqrt{n}}\right)$ .  $A$ 's preferences are given by  $u_A(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$ , and  $B$ 's preferences by  $u_B(x_1, x_2) = \sqrt{x_1} + x_2$ . Determine the Walrasian equilibrium prices and the equilibrium allocations. What happens in the limit when  $n$  grows without bound?

6. Consumers  $A$  and  $B$  have endowments  $\omega_A = (2, 0)$  and  $\omega_B = (1, 3)$ . The preferences are given by

$$u_A(x, y) = \sqrt{x} + \sqrt{y}$$

$$u_B(x, y) = x^3 - 9x^2 + 15x + y^3 - 9y^2 + 15y$$

Show that  $p = (1, 1)$  is a Walrasian equilibrium. Show that equilibrium allocations are not Pareto-optimal. Why is this?