Exercise set 1.

1. Put up the differential equations that determine utility functions such that they exhibit *a*) constant absolute risk aversion, and *b*) constant relative risk aversion, and proceed to solve the equations.

2. Consider a lottery F_1 and, by integrating $\int_{\underline{x}}^{\bar{x}} u(x) f_1(x) dx$ partially twice, derive an expression which features the second derivative of u; f_1 is the density of F_1. Then derive conditions guaranteeing that any risk averse agent (= for all concave utility functions) prefers lottery F_1 to lottery F_2, any risk neutral agent is indifferent between the lotteries, and any risk loving agent prefers F_2 to F_1.

3. Calculate the mean and variance of the following lotteries $F_1(x) = \begin{cases} 0 \text{ if } x < 0 \\ 0.25 \text{ if } x \leq 0 \\ 0.25 \text{ if } x < 2 \\ 0.5 \text{ if } x \leq 2 \\ 0.5 \text{ if } x < 4 \\ 0.75 \text{ if } x \leq 4 \\ 0.75 \text{ if } x < 6 \\ 1 \text{ if } x \leq 6 \end{cases}$ and $F_2(x) = \begin{cases} 0 \text{ if } x < 0.1 \\ 1/3 \text{ if } x \leq 0.1 \\ 1/3 \text{ if } x < 3 \\ 2/3 \text{ if } x \leq 3 \\ 2/3 \text{ if } x < 5.9 \\ 1 \text{ if } x \leq 5.9 \end{cases}$. Assume that an agent has utility function $u(x) = \begin{cases} 2x \text{ if } x \leq 3 \\ 3 + x \text{ if } x > 3 \end{cases}$, and show that the lottery with greater variance SOSD the lottery with the smaller variance.

4. Determine the Forster-Hart risk measure for lotteries $F_1(x) = \begin{cases} 0 \text{ if } x < -1 \\ 0.25 \text{ if } x \leq -1 \\ 0.25 \text{ if } x < 2 \\ 0.5 \text{ if } x \leq 2 \\ 0.5 \text{ if } x < 4 \\ 0.75 \text{ if } x \leq 4 \\ 0.75 \text{ if } x < 6 \\ 1 \text{ if } x \leq 6 \end{cases}$ and $F_2(x) = \begin{cases} 0 \text{ if } x < -1 \\ 1/3 \text{ if } x \leq -1 \\ 1/3 \text{ if } x < 3 \\ 2/3 \text{ if } x \leq 3 \\ 2/3 \text{ if } x < 5 \\ 1 \text{ if } x \leq 5 \end{cases}$.

5. Construct a utility function such that a expected utility maximiser with wealth w is willing to participate in a state lottery which costs one euro and with probability $\frac{1}{10^7}$ returns two million euros, and at the same time is willing to buy full insurance at actuarially fair price to insure his/her house the value which is half a million euros.

6. There is a risk averse DM with von Neumann-Morgenstern preferences and Bernoulli utility u. His/her initial wealth is w. There is a safe asset with certain unit return, and a risky asset with return r_i with probability p_i, $i \in \{1, ..., n\}$. The DM decides how much of his initial wealth w to put into a risky asset. If amount α is put into a risky asset then the final wealth is given by $w - \alpha + (1 + r_i)\,\alpha = w + r_i\alpha$ under outcome r_i. What happens to the amount invested in the risky asset as wealth changes? Use risk aversion, say DARA, as criterion.