General equilibrium theory further results

Sixth lecture

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Example

Let $\{\omega_n\}$ be a sequence of endowments such that $\omega_n \longrightarrow \omega$.
Let (p_n, x_n, y_n) be the corresponding sequence of Walrasian equilibrium prices, demands and production plans.
Assume that $(p_n, x_n, y_n) \longrightarrow (p, x, y)$ and that p is strictly positive.
Then (p, x, y) constitutes a Walrasian equilibrium at ω.
Going to the limit shows market clearing

$$\sum x_n^h \leq \sum \omega_n^h + \sum y_n^f$$

and profit maximisation letting $\hat{y}$ some other production plan

$$p_n y_n^f \geq p_n \hat{y}$$

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Example

Similarly it is clear that budget balance is respected

$$p_n x_n^h \leq p_n \omega_n^h + \sum \theta^{fh} p_n y_n^f$$

That x^h maximises utility is more complicated.
First show that if $\hat{x} \succ_h x^h$ then $p\hat{x} \geq p\omega^h + \sum \theta^{fh} p y^f$. There must be N such that if $n \geq N$ then $\hat{x} \succ_h x_n^h$ and thus $p_n \hat{x} \geq p_n \omega_n^h + \sum \theta^{fh} p_n y_n^f$ and then go to limit.

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Example

Next show that if $\hat{x} \succ_h x^h$ then $p\hat{x} > p\omega^h + \sum \theta^{fh} py^f$. If this is not the case then $p\hat{x} = p\omega^h + \sum \theta^{fh} py^f$. We know that $px^h \geq 0$, and then also $p\omega^h + \sum \theta^{fh} py^f \geq 0$. If $p\omega^h + \sum \theta^{fh} py^f = 0$ then the only bundle h can by is such that all entries are zero. As $\hat{x} \succ_h x^h = 0$ then $\hat{x} \neq 0$. As prices are strictly positive $px \hat{>} 0$ which is a contradiction.

Thus, we know that $p\omega^h + \sum \theta^{fh} py^f > 0$. Consider $\alpha\hat{x}$ where $\alpha < 1$ but which, by continuity of preferences, satisfies $\alpha\hat{x} \succ_h x^h$. But now

$$p\alpha\hat{x} < p\omega^h + \sum \theta^{fh} py^f$$

which is a contradiction.

Example

There are two agents with preferences $u_1(x_1, y_1) = x_1 + f_1(y_1)$ and $u_2(x_2, y_2) = y_2 + f_2(x_1)$ and endowments $(\tilde{x}, 0)$ and $(0, \tilde{y})$.
$f_i(z) = \alpha z - \frac{1}{2}z^2$ for $z \leq \alpha$.
Normalise prices $(p_x, p_y) = (1, p)$.
Assume that both agents' demands are interior; then marginal rates of substitution equal p

$$\alpha - y_1 = p$$

$$\frac{1}{\alpha - x_2} = p$$

Example

Agent 1's demand for good y is given by $Y_1(p) = \alpha - p$ and agent 2's demand for good x is given by $X_2(p) = \alpha - \frac{1}{p}$.
Using the budget constraint $Y_2(p) = \tilde{y} - \frac{\alpha}{p} + \frac{1}{p^2}$.
In equilibrium excess demand for good y must be zero or
$\alpha - p + \tilde{y} - \frac{\alpha}{p} + \frac{1}{p^2} - \tilde{y} = 0$ which is equivalent to
$p^3 - \alpha p^2 + \alpha p - 1 = 0$.
One immediately sees that $p = 1$ is a solution.
Factoring we have $(p-1)\left(p^2 + (1-\alpha) + 1\right) = 0$ and the other solutions are $\frac{-(1-\alpha)+\sqrt{(1-\alpha)^2-4}}{2} = r$ and $\frac{1}{r}$ if $\alpha > 3$.

Example

There are two agents with two goods. Both have preferences $u(x_1, x_2) = min\{x_1, x_2\}$. Endowments are given by $\omega_1 = (1, 0)$ and $\omega_2 = (0, 1)$. For any prices $p = (p_1, p_2)$, $p_1 + p_2 = 1$ agent 1 has wealth p_1 and agent 2 wealth p_2.The Marshalian demands at prices $p = (p_1, p_2)$ are then $x_1(p) = (p_1, p_1)$ and $x_2(p) = (p_2, p_2)$. Any prices constitute a Walrasian equilibrium. If $p_1 = 0$ there are other utility maximising choises.

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- A model is useful only if its equilibrium has some kind of uniqueness properties.
- The Walrasian equilibrium is not generally unique.
- The conditions that guarantee uniqueness are very stringent, i.e., not very plausible.
- Uniqueness is guaranteed, for instance, if all the goods are gross substitutes: In a differentiable case $\frac{\partial z_i}{\partial p_j} > 0$ for all $i \neq j$.
- Conditions for local uniqueness are of interest then.

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- Let us think of the economy parameterised by the endowments $\omega \in \mathbb{R}^{HL}_{++}$, i.e., the preferences are fixed in all comparative statics exercises.
- Since the excess demand correspondence z satisfies the Walras's law we normalise the price vector so that $p_L = 1$, and focus on the first $L-1$ components of z which can be thought as a mapping $z : \mathbb{R}^{L-1}_{++} \times \mathbb{R}^{HL}_{++} \longmapsto \mathbb{R}^{L-1}_{++}$.

Definition

Assume that $z(p,\omega) = 0$. If $D_p z(p,\omega)$ has rank $L-1$ the price p is regular. An economy parametrised by ω is regular if $D_p z(p,\omega)$ has rank $L-1$ for all $p \in \mathbb{R}^{L-1}_{++}$ such that $z(p,\omega) = 0$.

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Definition

Equilibrium price $p \in \mathbb{R}^{L-1}_{++}$ is locally unique if there exists an open set $N \ni p$ such that for all $p' \neq p$ and $p' \in N$ the excess demand is non-zero or $z(p', \omega) \neq 0$.

Theorem

A regular price $p \in \mathbb{R}^{L-1}_{++}$ such that $z(p, \omega) = 0$ is locally unique.

Proof.

Remember that the endowments are fixed. As $D_p z(p, \omega)$ has rank $L - 1$ we can infer the local uniqueness of p by the inverse function theorem applied to the mapping $z : \mathbb{R}^{L-1}_{++} \longmapsto \mathbb{R}^{L-1}_{++}$. □

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Theorem

Assume that an exchange economy is regular. Then there is a finite number of equilibria.

Proof.

Because the excess demand goes to infinity if any price is zero it must be the case that in equilibrium $\frac{1}{r} < p_l < r$ for some $r > 1$ and for every $l \in \mathscr{L}$. As z is upper hemi-continuous the set of equilibrium price vectors is compact. But a compact discrete set must be finite. □

Theorem

Exchange economies are generically regular, i.e., the set of non-regular economies is closed and of Lebesgue-measure zero.

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- A natural question concers all the empirical implications of the theory. A rather disappointing result is given by Sonnenschein-Mantel-Debreu theorem.

- Let us assume that $p \in int\triangle$. We say that an exchange economy $\mathscr{E} = ((u_h, \omega_h)_{h\in\mathscr{H}})$ generates excess demand $z : int\triangle \longmapsto \mathbb{R}^L$ in $\triangle_\varepsilon = \{p \in int\triangle : p_l \geq \varepsilon\, l \in \mathscr{L}\}$ if for all $p \in \triangle_\varepsilon$the following holds

$$\sum_{h\in\mathscr{H}} (x_h(p) - \omega_h) = z(p)$$

Theorem

Let $z : int\triangle \longmapsto \mathbb{R}^L$ be continuous and satisfy Walras's law. For every $\varepsilon > 0$ there exists and exchange economy $\mathscr{E} = ((u_h, \omega_h)_{h\in\mathscr{H}})$ that generates z in $\triangle_\varepsilon$.

Proof.

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- This is interpreted to mean that the theory does not provide any empirically testable implications.
- The view is typical but not entirely correct.
- Brown and Matzkin (1996) study what can be said if both prices and endowments are observable.

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Theorem

There exist price-endowment pairs $(p, (\omega_h)_{h \in \mathscr{H}})$ *and* $(p', (\omega'_h)_{h \in \mathscr{H}})$ *such that there are no monotone preferences* $(u_h)_{h \in \mathscr{H}}$ *such that* p *is a Walrasian equilibrium in economy* $\mathscr{E} = ((u_h, \omega_h)_{h \in \mathscr{H}})$ *and* p' *is a Walrasian equilibrium in economy* $\mathscr{E}' = ((u_h, \omega'_h)_{h \in \mathscr{H}})$.