FDPE Microeconomics 1: Decision theory
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Problem Set 2

1. Let concumer's preferences be represented by a utility function u reflecting constant elasticity of subsitution (CES)

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}, \text{ where } \rho \in (0,1)$$

Let product prices be p_1 and p_2 and income w.

   (a) Construct a Lagrangean that reflects the consumer's optimization problem.
   (b) Find the first order conditions for optimality.
   (c) Identify the Marshallian demand function.
   (d) Construct the indirect utility function.
   (e) Identify the Hicksian demand function.
   (f) Construct the expedinture function

2. Let v be a consumer's indirect utility function. Show, by using Roy's identity, that the Marshallian demand remains intact for any monotonic transformation of v. Can you say the same concerning the expenditure function e and Shephard's Lemma?

3. In a two good case, let consumer's wealth w be derived from selling her intial endowments $\omega_1, \omega_2 \geq 0$ with prices $p_1, p_2 \geq 0$. Then her budget line is characterized by the inequality

$$p_1 x_1 + p_2 x_2 \leq p_1 \omega_1 + p_2 \omega_2.$$

Let $x(p)$ represent the optimal demand of the consumer and $v(p)$ the associated indirect utility under $p = (p_1, p_2)$. Her "excess demand" of good ℓ is then $x_\ell(p) - \omega_\ell$. Use the envelope argument to show that

$$\frac{\partial v(p) / \partial p_1}{\partial v(p) / \partial p_2} = \frac{x_1(p) - \omega_1}{x_2(p) - \omega_2}.$$

4. Let preferences be represented by a utility function

$$u(x) = \sum_{i=1}^{L} \beta_i \ln(x_i).$$

Fixing prices $(p_1, ..., p_L)$ and income w, determine the Marshallian demand. Relate your findings to the Cobb-Douglas example in the lecture notes.

5. Suppose that $x_1(p, w)$ and $x_2(p, w)$ have the same income elasticities at (p, w). Show that $\partial x_1(p, w) / \partial p_2 = \partial x_2(p, w) / \partial p_1$.

6. Preferences are said to be additively separable if they can be represented by a utility function of the form

$$u(x) = \sum_{i=1}^{L} u_i(x_i).$$

   (a) Show that all goods are normal if $u_i' > 0$ and $u_i'' < 0$ for all i

   (b) Show that, for all i, j, k,

$$\frac{\partial x_i(p, w) / \partial p_k}{\partial x_j(p, w) / \partial p_k} = \frac{\partial x_i(p, w) / \partial w}{\partial x_j(p, w) / \partial w}.$$

   (c) Suppose that $u_1(x_1) = x_1$ and let $p_1 = 1$. What happens to the consumption of goods $2, 3, ..., L$ when w becomes large ($\geq \max\{p_2, ..., p_L\}$)?

7. Show that if the preferences are homothetic, then:

   (a) The Marshallian demand function is multiplicatively separable in prices and income such that $x_i(p, w) = \phi(w) x_i(p, 1)$, for some increasing ϕ

   (b) The expenditure function is multiplicatively separable in prices and utility such that $e_i(p, \bar{u}) = \eta(\bar{u}) e_i(p, 1)$, for some increasing η.

   (c) The income elasticity of Marshallian demand for every good is equal to one.

8. Can all the goods be inferior at the point (p, w)?

9. Suppose that the Marshallian demand of the consumer is

$$x_1(p, w) = \frac{w}{p_1 + 2p_2} \text{ and } x_2(p, w) = \frac{2w}{p_1 + 2p_2}$$

Is there a utility function reflecting monotonic, convex preferences that gives rise to this demand under all (p, w)?

10. An infinitely lived agent owns one unit of a commodity that she consumes over her lifetime (dsitrbutes over periods $0, 1, 2, ...$ without costs). Her lifetime utility is given by

$$u(x_0, x_1, x_2, ...) = \sum_{t=0}^{\infty} \delta^t \ln(x_t).$$

Identify the optimal consumption stream.