FDPE Microeconomics 1: Decision theory
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Problem Set 1

1. Prove the propositions stated in the lecture notes:
   - Let the choice structure $\mathcal{A}$ include all two or three element subsets of X. If $c(\cdot)$ satisfies WARP on $\mathcal{A}$, then the induced revealed preference relation $\succsim^*$is rational.
   - If $\succsim$ is a rational preference relation, then the choice function $c^*(\cdot, \succsim)$ induced by $\succsim$ satisfies WARP

2. Show that if c is singe valued, then WARP=IIA. Further, show that the resulting revealed preference relation $\succsim^*$ is *strict.*

3. Amartya Sen's two conditions on a non-empty choice rule c :
   - Condition α : if $x \in c(A)$ and $x \in B \subseteq A$, then $x \in c(B)$.
   - Condition β : if $x, y \in c(B)$ and $B \subseteq A$, $y \in c(A)$, then $x \in c(A)$.

   Interpret the conditions. Show that α and β together are equivalent with WARP.

4. Show that the definition of continuous preferences given in the class is equivalent with the following: Preferences $\succsim$ on X are continuous if whenever $x \succ y$, there are open neighborhoods B_x and B_y around x and y respectively such that for any $x' \in B_x$ and $y' \in B_y$ also $x' \succ y'$.

5. Show that if preferences $\succsim$ are represented by a continuous utility function u, then $\succsim$ is continuous (hint: employ the definition of continuity stated in the previous problem).

6. Show that if preferences $\succsim$ on $\mathbb{R}^L_+$ satisfy continuity, and $x \succsim z \succsim y$, then there is a $w \in \mathbb{R}^L_+$ in the line segment connecting x and y such that $w \sim z$.

7. Show that the preferences $\succsim$ on $X = \mathbb{R}^L_+$ are convex if and only if a utility function u representing them is quasi-concave.

8. Let continuous preferences on $\mathbb{R}^2_+$ be represented by a utility function of the form $u(x_1, x_2) = a_1x_1 + a_2x_2$, for some $a_1, a_2 > 0$. Show that then $\succsim$ satisfy the following conditions:

- Additivity: $(x_1, x_2) \succsim (y_1, y_2)$ implies $(x_1 + d_1, x_2 + d_2) \succsim (y_1 + d_1, y_2 + d_2)$, for any numbers d_1, d_2
- Monotonicity: If $x_1 \geq y_1$ and $x_2 \geq y_2$, then $(x_1, x_2) \succsim (y_1, y_2)$.
- Continuity

9. The government finances public expenditure of magnitude g by collecting taxes. In this question, you are invited to think about the optimal ways of collecting taxes. Suppose that there are goods x and y. The government can finance g by choosing either a tax on income t_w or by taxing consumption of good x by rate t_x. The government budget constraint for the two cases reads: $t_w w = g$ and $t_x x\,(p_x, p_y, t_x) = g$. Show that the consumer prefers an income tax in this case.

10. Let u and v both be representation of preferences $\succsim$ on X. Show that a function $g : \mathbb{R} \to \mathbb{R}$ such that $v(x) = g(u(x))$ for all $x \in X$ must be *strictly* increasing (in the domain $u(X)$).

11. Preferences $\succsim$ on $\mathbb{R}^L_+$ are *homothetic* if $x \succsim y$ implies $\lambda x \succsim \lambda y$, for all $\lambda > 0$ (where $\lambda x = (\lambda x_1, ..., \lambda x_L)$). Function $f : \mathbb{R}^L_+ \to \mathbb{R}$ is *homogenous* (of degree 1) if $f(\lambda x) = \lambda f(x)$. Show that continuous function u represents cotinuous homothetic preferences if and only if u is homogenous.

12. Let preferences $\succsim$ on $\mathbb{R}^2_+$ be of Leontieff form: $(x_1, x_2) \succsim (y_1, y_2)$ if $\min\{x_1, x_2\} \geq \min\{y_1, y_2\}$. Find a utility representation for these preferences. Identify the Marshallian demand function $x(p, w)$ and the indirect utility function.