Lecture notes 3b: Demand, welfare, and price changes

Hannu Vartiainen
HECER

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Income and substitution effects

- We are ultimately interested in the relationship between the environment and the consumer behavior, i.e. how the changes in income and prices modify her consumption
- Instinctively, we tend to think that a decrease of a good's price will increase its demand => "Law of Demand"
- To verify whether this holds true, we need to decompose the effects of a price change into **substitution effect** and **income effect**

- Substitution effect: relatively cheaper goods become more attractive
- Income effect: increased income permits optimization, cannot say much about the direction for a particular good
  - *Normal* good's demand increases as income increases
  - *Inferior* good's demand decreases as income increases
- "Local" properties; may depend on the current level of prices, income and consumption

- Total effect of a price change on goods demand?

Proposition (Slutsky decomposition)

Let $\bar{u}$ be value of the indirect utility at (p, w). The effect of price change of good ℓ on good k can be written

$$\frac{\partial x_k (p, w)}{\partial p_\ell} = \underbrace{\frac{\partial h_k (p, \bar{u})}{\partial p_\ell}}_{\text{Substit. eff.}} - \underbrace{x_\ell (p, w) \frac{\partial x_k (p, w)}{\partial w}}_{\text{Income eff.}}$$

Proof.

By duality, for all p,

$$h(p,\bar{u}) = x(p, e(p,\bar{u}))$$

By differentiating both sides,

$$\frac{\partial h_k(p,\bar{u})}{\partial p_\ell} = \frac{\partial x_k(p, e(p,\bar{u}))}{\partial p_\ell} + \frac{\partial e(p,\bar{u})}{\partial p_\ell}\frac{\partial x_k(p, e(p,\bar{u}))}{\partial w}$$

Rearranging,

$$\frac{\partial x_k(p, e(p,\bar{u}))}{\partial p_\ell} = \frac{\partial h_k(p,\bar{u})}{\partial p_\ell} - \frac{\partial e(p,\bar{u})}{\partial p_\ell}\frac{\partial x_k(p, e(p,\bar{u}))}{\partial w}$$

□

Proof.

(cont.) Using Shephard's Lemma,

$$\frac{\partial x_k\left(p, e\left(p, \bar{u}\right)\right)}{\partial p_\ell} = \frac{\partial h_k\left(p, \bar{u}\right)}{\partial p_\ell} - h_\ell\left(p, \bar{u}\right)\frac{\partial x_k\left(p, e\left(p, \bar{u}\right)\right)}{\partial w}$$

Noting that, by duality, $h\left(p, \bar{u}\right) = x\left(p, e\left(p, \bar{u}\right)\right)$ and $e\left(p, \bar{u}\right) = w$, the result follows. □

- In particular,

$$\frac{\partial x_\ell(p,w)}{\partial p_\ell} = \frac{\partial h_\ell(p,\bar{u})}{\partial p_\ell} - x_\ell(p,w)\frac{\partial x_\ell(p,w)}{\partial w}$$

- Since e is a concave function in p, and by Shephard's Lemma

$$h_\ell(p,\bar{u}) = \frac{\partial e(p,\bar{u})}{\partial p_\ell}$$

  it follows that $\partial h_\ell(p,\bar{u})/\partial p_\ell = \partial^2 e(p,\bar{u})/(\partial p_\ell)^2$ must be nonpositive

- Classic assumption/result in economics: price increase decreases demand => "Law of Demand"
- A good is normal if $\partial x_\ell (p, w) / \partial w$ is nonnegative and inferior otherwise

Proposition ("Modern" Law of Demand)

If a good is normal, then its demand decreases as its price increases. If the good's demand increases as its price decreases (= "Giffen good"), then the good must be inferior.

- Hicksian demand is not directly observable but by the Slutsky decomposition it's derivative is!

$$\frac{\partial h_k\left(p,\bar{u}\right)}{\partial p_\ell}=\frac{\partial x_k\left(p,w\right)}{\partial p_\ell}+x_\ell\left(p,w\right)\frac{\partial x_k\left(p,w\right)}{\partial w}$$

(evrything in the left hand side is in principle observable)

- By Shephard's Lemma we know that rational preferences imply certain properties on the expedinture function and hence on the Hicksian demand
$=>$ an additional empirical regularity conditions on the the Slutsky substitution matrix $S(p,w)$ consisting of terms

$$s_{k\ell}(p,w)=\frac{\partial h_k\left(p,\bar{u}\right)}{\partial p_\ell},\text{ for all }k,\ell=1,...,L$$

Proposition

The substitution matrix $S(p, w)$ is symmetric and negative semidefinite.

Proof.

By Shephard's Lemma, $h_\ell(p, \bar{u}) = \partial e(p, \bar{u}) / \partial p_\ell$. That $S(p, w)$ is negative semidefinite follows from the concavity of e. Symmetry in turn is a consequence of the fact that

$$\frac{\partial h_k(p, \bar{u})}{\partial p_\ell} = \frac{\partial^2 e(p, \bar{u})}{\partial p_k \partial p_\ell} = \frac{\partial^2 e(p, \bar{u})}{\partial p_\ell \partial p_k} = \frac{\partial h_k(p, \bar{u})}{\partial p_\ell}$$

□

- In particular, there are *no* preferences that rationalize observed demand if the derived substitution matrix is not symmetric

Money metric utility function

- The expenditure function $e(p, \bar{u})$ specifies the least amount of budget that is needed to achieve the utility $\bar{u}$ under prices p, for any $\bar{u}$
- Then, for any given prices p, the function $e(p, u(\cdot))$ is actually a utility function representing the same preferences as u
- To see this, note that $e(p, u(x)) \geq e(p, u(x'))$ iff $u(x) \geq u(x')$; the higher indifference curve the consumer wants to achieve, the bigger budget he needs *under given p*
- $e(p, u(\cdot))$ is referred as a **money metric utility function**,and can, at least in principle, recovered by the repeated decision problems of the type: "given prices p, would your rather choose income w or bundle x"
- **But:** $e(p, u(x))$ and $e(p', u(x'))$ no longer comparable under distinct p and p'

Price changes and welfare

- How much better or worse off is the consumer if the price of good ℓ changes from p_ℓ to p'_ℓ? (assume $p'_\ell > p_\ell$ and denote $p = (p_\ell, p_{-\ell})$, $p' = (p'_\ell, p_{-\ell})$)
- A natural candidate: the difference between the indirect utility functions at the two points $v(p, w)$ and $v(p', w)$
- ...but v is not observable!

- Classic approach: **consumer surplus** CS

$$CS = \int_{p}^{p'} x_\ell(s, w)ds$$

- Good: observable, bad: not clear what CS *means*
- But works well under quasi-linear preferences

Example

Let consumer's preferences be represented in quasi-linear form $u(x, m) = f(x) + m$ for $x \in \mathbb{R}^L_+$ and $m \in \mathbb{R}$. Then $v(p, w) = f(x(p, w)) + (w - \sum_\ell p_\ell x(p, w))$. Since $\partial v(p, w)/\partial w = 1$, by Roy's Identity,

$$x_\ell(p, w) = \frac{\partial v(p, w)}{\partial p_\ell}.$$

Thus

$$v(p, w) - v(p', w) = \int_p^{p'} x_\ell(s, w) ds = CS$$

- Another candidate: ask how much the consumer should be compensated w to neutralize the price change - a measure of of the impact of the change
- The first approach: evaluate the needed compensation at the level of utility of the consumer prior to the change
- **Compensating variation** CV specifies how much wealth the consumer needs to attain the same maximum utility under p' than she received under p

$$CV = e(p, v(p, w)) - e(p', v(p, w))$$

- That is, CV tells us how much we will have to compensate our consumer to have her stay on the same indifference curve despite the price change

- **Equivalent variation** EV gives the change in the expenditure that would be required at the original prices to have the same ("equivalent") effect on consumer as the price change $\in$

$$EV = e(p, v(p', w)) - e(p', v(p', w))$$

- That is, EV tells us how much more money the consumer would have needed yesterday to be as well off as she is today

- Using Shephard's lemma

$$CV = \int_{p'}^{p} \frac{de(s, v(p, w))}{ds} ds = \int_{p'}^{p} h(s, v(p, w)) ds$$

and

$$EV = \int_{p}^{p'} \frac{de(s, v(p', w))}{ds} ds = \int_{p}^{p'} h_{\ell}(s, v(p', w)) ds$$

- But, again, since v is not observable, $e(p, v(p', w))$ is not observable

- Note that $\{h(\cdot, v(s,w))\}_{s\in[p,p']}$ constitutes a family of non-crossing Hicksian demand functions
- If ℓ is a normal good, by the Slutsky equation,

$$\frac{\partial h_\ell\left(p, v(p,w)\right))}{\partial p_\ell} < \frac{\partial x_\ell\left(p,w\right)}{\partial p_\ell}$$

- Since, by duality

$$h_\ell(p, v(p,w)) = x_\ell(p,w) \quad \text{and} \quad h_\ell(p', v(p',w)) = x_\ell(p',w)$$

we conclude that

$$h_\ell(s, v(p,w)) \geq x_\ell(s,w) \geq h_\ell(s, v(p',w)), \quad \text{for all } s \in [p,p']$$

- Finally

$$\int_p^{p'} h_\ell(s, v(p,w))ds \geq \int_p^{p'} x_\ell(s,w)ds \geq \int_p^{p'} h_\ell(s, v(p',w))ds$$

implying that $EV \geq CS \geq CV$

Example

(cont) In the quasi-linear utility case, the Hicsian demand of goods 1, ..., L is independent of the wealth level, i.e.

$$h(s, v(p, w)) = h(s, v(p', w)), \quad \text{for all } s \in [p, p'].$$

Thus

$$\int_p^{p'} h_\ell(s, v(p, w))ds = \int_p^{p'} x_\ell(s, w)ds = \int_p^{p'} h_\ell(s, v(p', w))ds$$

implying that $EV = CS = CV$.

Revealed consumer choice

- This far our study of consumer behavior has taken as the primitive "well behaved" preferences and we have examined the restrictions they impose on observable choices (symmetric negative semidefinite Slutsky matrix)
- But can we really see the Marhallian demand function?
- How does this model relate to our earlier discussion revealed preferences?

- T different price/wage combinations $(p^1, w^1), ..., (p^T, w^T)$ induce T decision problems $B(p^1, w^1), ..., B(p^T, w^T)$
- A utility function u **rationalizes** the observed demands $x(p^1, w^1), ..., x(p^T, w^T)$ if $u(x(p^t, w^t)) \geq u(x')$ for all $x' \in B(p^t, w^t)$ for all $t = 0, ..., T$
- But note that complete indifference would rationalize any choices => WARP has no not bite
- Assuming that u reflects locally nonsatistaed preferences we can say more:
  - If $\sum p^t_\ell x_\ell \leq \sum p^t_\ell x_\ell(p^t, w^t)$ then $u(x(p^t, w^t)) \geq u(x)$
  - If $\sum p^t_\ell x_\ell < \sum p^t_\ell x_\ell(p^t, w^t)$ then $u(x(p^t, w^t)) > u(x)$
- Thus
  - If $\sum p^t_\ell x_\ell \leq \sum p^t_\ell x_\ell(p^t, w^t)$ then $x(p^t, w^t)$ is **directly revealed preferred** to x
  - If $\sum p^t_\ell x_\ell < \sum p^t_\ell x_\ell(p^t, w^t)$ then $x(p^t, w^t)$ is **strictly directly revealed preferred** to x

- If, for some indexation of the observations, it holds that $x(p^t, w^t)$ is directly revealed preferred to $x(p^{t+1}, w^{t+1})$ for all 1, ..., $T' - 1$, then we say that $x(p^1, w^1)$ is **revealed preferred** to $x(p^{T'}, w^{T'})$

Axiom (Generalized Axiom of Revealed Preference, GARP)

If $x(p, w)$ is revealed preferred to $x(p', w')$, then $x(p', w')$ is not **directly** *revealed preferred to $x(p, w)$*

- The next result states that *if* the data can be rationalized by locally nonsatiated preferences, *then* it can be rationalized by preferences meeting all the other standard assumption (monotnic, convex, continuous)
  => the "additional" assumption cannot be rejected without rejecting local nonsatiation!

Proposition (Afriat)

The following conditions are equivalent

1. *There is a locally nonsatiated utility function that rationalizes the (finite) data*
2. *The satisfies GARP*
3. *There is a locally concave, monotonic utility function that rationalizes the (finite) data*