FDPE Microeconomics 1: Decision Theory

Hannu Vartiainen
University of Helsinki

Fall 2015

Building blocks of (micro)economic theory

- **Methodological individualism**: a principle according to which social phenomena can only be understood by examining how they result from actions of individual agents
- **Rationality:** agents make decisions *as if* they maximize some preference ordering
- **Equilibrium:** agents' decisions are tied together under the hypothesis of multisided simultaneous optimization and the agents' knowldge on this
- Models are, as in any honest scientific enterprise, formal
  - Permits clear insight
  - Makes models comparable and integrable
  - Rules out faulty logic
  - Comparative static exercises
  - Facilitates testing the model

- The only thing that is even in principle observable from the agent is his behavior
- What does observed (economic) behavior tell us about the decision maker (DM)? => **her preferences**
- Obs.: "utility" cannot be observed!
- Observations without a model meaningless - finding the right model crucial

From choice to preferences

- What can we deduce from the observed behavior?
- We know:

1. **Universe of choosable alternatives** X; the set of possible objects that might be chosen
   - Imaginable dishes
   - $\mathbb{R}^n_+$
2. **Feasible set** $A \subseteq X$; given by external conditions
   - Manu in a restaurant
   - Budget set $B(p, m) = \left\{ x \in \mathbb{R}^L_+ : \sum_{l=1}^{L} p_l \cdot x_l \leq m \right\}$ with L commodities, prices $p_0, ..., p_L$ and budget m
3. **Choice rule** c that assigns $c(A)$ to each possible A

- Why separate A and X?

Choice rule

- How is choice made when A is given?
- We need a behavioral assumption, reflecting "rationality", that ties together choices in differenct contexts
- Let $\mathcal{A}$ denote the collection of all possible feasible sets in X, call $\mathcal{A}$ a **choice structure** with typical element A called a **choice problem**
- The choice rule c assigns to each set A in the choice structure $\mathcal{A}$ a subset of the elements in A, i.e. $c(A) \subseteq A$, with the interpretation that $c(A)$ constitutes the elements in A that are **choosable**
- $c(A)$ is the **only** information that we could ever get from the DM in the choice context $A \in \mathcal{A}$

Behavioral assumption

- In economics, we apply Occam's razor by assuming that c reflects "rationality"
- What would rationality imply for $c(A)$?

Axiom (Weak Axiom of Revealed Preference, WARP)

If, for any $A, B \in \mathcal{A}$, there are x, y such that $x, y \in A \cap B$, $x \in c(A)$, and $y \in c(B)$, then $x \in c(B)$

- If the DM chooses x over y, then being informed of the presence of z does make not her to choose y over x

- If, additionally, we require that c is single valued, WARP reduces to what is known as the **Independence of Irrelevant Alternatives**: if $c(A) \in A \cap B$, then $c(A) = c(A \cap B)$
- Removing non-chosen outcomes will not affect the choice; IIA is a **consistency** property
- Our aim is to show that if the agent chooses according to WARP, then he behaves **as if** he has rational preferences that he maximizes (and conversely)

What are preferences?

- Preferences reflect the summary of all judgements of the agent, how he compares distinct alternatives against one another
- Independent of the context A, i.e. desirability does not depend on feasibility
- **Preference relation** $\succsim$ is a binary relation, a subset of $X \times X$, written for convenience $x \succsim y$ when $(x, y) \in \succsim$
- Interpretation of $\succsim$: in DM's opinion x is at least as good as y iff $x \succsim y$
- In this language "x is much better than y" cannot be expressed
- Other binary relations derived from $\succsim$:
  - Indifference part: $x \sim y$ if $x \succsim y$ and $y \succsim x$
  - Strict part: $x \succ y$ if $x \succsim y$ and not $y \succsim x$

Rational preferences

Axiom (Completeness)

For all $x, y \in X$ *either* $x \succsim y$ *or* $y \succsim x$

Axiom (Transitivity)

For all $x, y, z \in X$, *if* $x \succsim y$ *and* $y \succsim z$, *then* $x \succsim z$

- Complete and transitive preferences are called **rational**
- Rationality thus means nothing but that the DM can **order** the alternatives
- Below, we may simplify exposition by also ruling out indifferences

Axiom (Strictness)

For all $x, y \in X$, *if* $x \succsim y$ *and* $y \succsim x$, *then* $x = y$

- Ratrionality rules out mioney pump (and vice versa!)

Example (Money pump)

The DM is willing to pay 1€ to replace an apple to banana, 1€ to replace an banana to orange, and 1€ to replace an orange to apple. Whenever, she has x at her hand, she is thus willing to pay 50c to replace it to something else. Soon, she is in financial troubles.

Example (Multi-attribute decisions)

Let agent's preferences concerning a cars x, y, and z depend on the price, reliability, and coolness. A car is preferred to anther if it is better in terms of two of the criteria. Let criteria based ranking be

Rank	Price	Reliability	Coolness
1.	x	y	z
2.	y	z	x
3.	z	x	y

By majority relation $x \succ y$, $y \succ z$, $z \succ x$. Hence no maximal choice exists.

- Given the observed choice function $c(\cdot)$, we can define the **revealed preference relation** $\succsim^*$:

$$x \succsim^* y \text{ if } x, y \in A \text{ and } x \in c(A), \text{ for some } A \in \mathcal{A}$$

- "$x \succsim^* y$" means "x is observationally at least as good as y" or "y is observationally not preferred to x"

Proposition

Let the choice structure $\mathcal{A}$ include all two or three element subsets of X. If $c(\cdot)$ satisfies WARP on $\mathcal{A}$, then the induced revealed preference relation $\succsim^*$ is rational

- That is, $\succsim^*$ **rationalizes** c if c meets WARP

- Why is the restriction on the sets in the previous proposition important?
- Example 1: $X = \{x, y, z\}$, $\mathcal{A} = \{\{x, y\}, \{y, x\}, \{x, z\}\}$
- Example2: As Ex. 1 but add X to $\mathcal{A}$

- To obtain the other direction, since rational preferences $\succsim$ put alternatives into an order, each subset A of X contains a unique $\succsim$ $-$maximal element denoted by $c^*(A, \succsim)$

Proposition

If $\succsim$ is a rational preference relation, then the choice function $c^*(\cdot, \succsim)$ induced by $\succsim$ satisfies WARP

Interpretation

- If the sample of observations is sufficiently rich ($\mathcal{A}$ includes all subsets of X with two or three elements), rationality (complete and transitive preferences) is **equivalent to** WARP
- Conversely, rejecting the (as if) rationality would imply rejection on WARP – plausible?
- Taking rational preferences as the starting point means that the analysis is based on (potentially) **observable** characterisitics of the decision maker (assuming WARP)
- In principle testable hypothesis; violation of WARP $=>$ not rational
- What about the converse, could WARP be generated via alternative decision making procedures?

Alternative approaches to decision making

- Psychological elements such as feelings, emotions, anxiety, excitement do **not** affect the rational choice theory as such: there is no reason why the preference relation $\succsim$ could not summarize the effect of these as well
- Psychological effects may have an impact if they affect the decision making procedures of the agent: **how** she deliberates and chooses
- Resulting models, which emphasize frictions implied by the procedure, reflect **bounded rationality**

Example (Satisficing)

(Herbert Simon): DM arranges the alternatives in A into an ordering, and starts checking the value of the candidates in this ordering one-by-one. The first alternative whose value exceeds a particular threshold value is chosen. If there is not element in A whose value exceeds the threhold, then final evaluated element in A is chosen.

- If the ordering in the list is the same across all $A \subseteq X$, the observed choice function c^* meets IIA, and is made **as if** there is a rational preference ordering that is maximized
- If the ordering in the list varies between $A \subseteq X$, the observed choice function c^* does **not** meet IIA, and cannot imitated by a rational choice model

- Hence, since IIA = WARP under single valued choice rule = rationalized by choices under strict rational preferences, choices under strict rational preferences **observationally indistinguishable** from satisificing behavior (Rubinstein and Salant)
- Satisficing one of the very few models of decision making that meet the IIA
- However, satisficing super sensitive to the underlying assumptions (how to choose listing order), and hence more complicated and arbitrary than rationality

Examples (Framing)

(Kahnemann and Tversky): An outbreak of a disease will cause 600 deaths. One of two emergency programs may be executed:

1. 400 people will die
2. with prob. $1/3$, no-one dies and with prob. $2/3$, all die

Another way to describe the decision problem:

1'. 200 people will be saved

2'. with prob. $1/3$, all will be saved and with prob. $2/3$, no-one will be saved

Experminetal subjects typically choose 2 and 1'

Utility representation

- Real utility or happiness, if it exists, is not used in nor required by economics models
- However, we often work with a **utility functions** for convenience: it can be easily manipulated, and it nicely summarizes the information contained in preferences
- Then the utility function **represents** preferences
- Is it OK to let a real-valued function to represent potentially complicated preferences over the choice set? What are we exactly assuming when taking this approach?

- We say that a utility function $u : X \to \mathbb{R}$ **represents preferences** $\succsim$ if it holds that

$$u(x) \geq u(y) \text{ if and only if } x \succsim y, \text{ for all } x, y \in X$$

- No additional interpretation associated to u, in particular, u does not reflect the level of satisfaction nor "happiness"

Proposition

If there exists a utility function representing $\succsim$, then $\succsim$ is rational

- Note: If u represents $\succsim$, then so does $f \circ u$ for **any** increasing $f : \mathbb{R} \to \mathbb{R}$
  $=>$ Utilities here do not have any interpretation as the level of satisfaction or "happiness"

- When the underlying environment is countable, one can always **construct** a utility function step-by-step, starting from a specific outcome and adding or substracting utility when moving upwards or downwards in preferences

Proposition

If the choice set X is countable and $\succsim$ is rational, then $\succsim$ has a utility representation.

Proof.

Enumarate the elements of X by $x_0, x_1, \ldots$ Choose $u(x_0) = 0$. Let $u(x_1) = 1$ if $x_1 \succ x_0$ and $u(x_1) = -1$ if $x_0 \succ x_1$. The remainder of the proof is by induction on the index k. For any $k = 1, 2, \ldots$, choose $u(x_k) = [\max_{x_k \succsim x_\ell, \ell < k} u(x_\ell) - \min_{x_\ell \succsim x_k, \ell < k} u(x_\ell)]/2$. □

- One can imagine noncountable situations where utility representation does exist: e.g. consumption of a single desirable good
- Does a utility representation always exist?

Example

Let preferences.on $X = [0,1] \times [0,1]$ be **Lexicographic** such that

$$(x_1, x_2) \succsim (y_1, y_2)$$
$$\text{if and only if}$$
$$x_1 \geq y_1 \text{ or } [x_1 = y_1 \text{ and } x_2 \geq y_2].$$

Assuming a representation u for these preferences leads to impossibility:
Suppose u represents preferences. Then
$u(a,1) > u(a,0) > u(b,1)$, for any $a, b \in [0,1]$ such that $a > b$.
For any a, choose a rational number $f(a)$ such that
$u(a,1) > f(a) > u(a,0)$. Then f is a strictly monotonic function from $[0,1]$ to the set of rational numbers, i.e. there is a 1-1 mapping from a continuum to a subset of rational numbers, a contradiction.

- Implication: further restrictions on the preference relation are needed
- Let $X = \mathbb{R}^L_+$, e.g. the set of commodity bundles.
- Define the **upper contour set** (or simply upper set) at x by

$$\succsim(x) = \{y \in X : y \succsim x\}$$

- Similarly, the **lower contour set** (or simply lower set) at x is given by

$$\precsim(x) = \{y \in X : x \succsim y\}$$

- and the **indifference set** at x is denoted by

$$I(x) = \{y \in X : x \succsim y \text{ and } y \succsim x\}$$

- The set $Y \subseteq X$ is **closed** if for all sequences $\{y_n\}$ such that $y_n \rightarrow y$ and $y_n \in Y$, we have $y \in Y$
- If $\precsim(x)$ and $\succsim(x)$ are closed, so is their intersection $I(x)$

- Note that a path from $y \in \succsim(x)$ to $z \in \precsim(x)$ passes through a point of indifference

Axiom (Continuity)

Preferences $\succsim$ are **continuous** *if, for all $x \in X$, the sets $\succsim(x)$ and $\precsim(x)$ are closed*

- If the agent strictly prefers x to y, and preferences are continuous, then a small perturbation of x (or y) does not affect the ranking
- The next result states that, in a consumer choice context, rational preferences have a utility function characterization under very general conditions

Theorem

Let $X = \mathbb{R}^L_+$. If $\succsim$ is rational and continuous, then there exists a utility function $u(\cdot)$ that represents $\succsim$.

Proof.

Let Y be a countable set that is a dense subset of X (e.g. rationals). Let v be the utility function on Y (such exists by the previous proposition). Choose $u(x) = \sup\{v(y) : y \in Y, x \succ y\}$, for all $x \in X$. We claim that $u(x) \geq u(y)$ if $x \succsim y$ for all $x, y \in X$. By the construction of u, $u(x) = u(y)$ if $x \sim y$. Let $x \succ y$. By the continuity of preferences, $\succsim (x)$ and $\precsim (y)$ are closed sets. By the transitivity of preferences, these two sets are disjoint. Since $X \backslash \succsim (x) \cup \precsim (y)$ is a nonempty open set, and hence contains an element of Y, there is $z_1 \in Y$ such that $x \succ z_1 \succ y$. Repeating the argument with respect to z_1 and y, there is $z_2 \in Y$ such that $x \succ z_1 \succ z_2 \succ y$. Since v is a utility function on Y, by the construction of u, $u(x) \geq v(z_1) > v(z_2) \geq u(y)$, as desired. □

- Does not require assumptions regarding tastes (convexity, monotonicity)
- Note: The existence of a utility function does not guarantee that there is an optimal decision even if the choice set is compact

Example

Let $X = [0, 1]$ with $x \succ 1$ and $x \succ y$ for all $x, y \in [0, 1)$ such that $x > y$.

Positive theory?

- Do people really maximize a numerical utility function?
- The model is "*as if*", does not require consciousness
- We have shown: if the DM adheres to the IIA, then she behaves as if she has a utility function
- Stability of the preferences

Utility and happiness

- Could utility functions be replaced with "happiness functions"?
- How can happiness be measured?
  - Nonlinear relationship between happiness and hormones
  - Requires a theory how the brain connects to the mind (who makes the decision) -> mind-body problem

- Problematic questioners
  - Language and norm dependence
  - The order of questions
  - Correlation with weather but not when the weather is pointed out
  - Meaning of life not evaluated

- National well being is often measured through GDP or equivalent
- Can happiness be measured by wealth?
  - Easterlin paradox
  - Stimulus effect
  - Keeping up with the Joneses